The role of double helicity-flip amplitudes in small-angle elastic pp-scattering

S. M. Troshin, N. E. Tyurin

Institute for High Energy Physics,
Protvino, Moscow Region, 142281 Russia

Abstract

We asses a role of the double helicity-flip amplitudes in small-angle elastic pp-scattering and obtain a new unitary bound for the double helicity-flip amplitude F_2 in elastic pp-scattering at small values of t on the basis of the U–matrix method of the s–channel unitarization.

PACS number(s): 13.60.Hb, 13.88.+e

Discussion of a role and magnitude of helicity-flip amplitudes in small-angle elastic scattering has a long history and is an important issue in the studies of the spin properties of diffraction. Recently an interest in accounting the contributions of single helicity-flip amplitudes becomes associated with CNI polarimetry related problems [1, 2, 3] as well. Bound for the single helicity-flip amplitude F_5 of elastic pp-scattering valid at finite energies has been derived in [1]. It corresponds to the asymptotic bound $cs\ln^3 s$ for the function $\hat{F}_5(s,0) \equiv [mF_5(s,t)/\sqrt{-t}]|_{t=0}$. Asymptotic unitarity bound valid in high energy limit obtained in [4] is stronger and it shows that $\hat{F}_5(s,0)$ cannot rise at $s \to \infty$ faster than $cs\ln^2 s$, i.e. this bound is similar to the Froissart-Martin bound for the helicity non-flip amplitudes. However, not only non-flip and single helicity-flip amplitudes can give contributions and affect the estimates and bounds for the analyzing power A_N. Double helicity-flip amplitudes can also contribute into A_N and their behavior at high energies is also important for the spin correlation parameters and total cross-section differences in experiments with two polarized beams available at RHIC nowadays.

The double helicity-flip amplitudes are usually neglected since they are supposed to be small in the whole region of momentum transfers. But this assumption is based merely on the technical simplification of the problem and is not valid at large momentum transfers in elastic pp-scattering where double-flip amplitudes can play an important role and fill up multiple-dip structure in differential cross-section providing correct description of the experimental data [5]. It is natural then to asses the role of double helicity-flip amplitudes at small and moderate values of t also. In this note we use unitarization method based on the U-matrix approach and obtain bounds for the amplitudes F_2 and F_4 which provide ground for the assumptions on their size and lead to the high-energy bounds for the cross-section difference $\Delta\sigma_T(s)$.

The method is based on the unitarity equation for helicity amplitudes of elastic pp-scattering. It should be noted here that there is no universal, generally accepted method to implement unitarity in high energy scattering. However, a choice of particular unitarization scheme is not completely a matter of taste. Long time ago the arguments based on analytical properties of the scattering amplitude were put forward [6] in favor of the rational form of unitarization. It was shown that this form of unitarization reproduced correct analytical properties of the scattering amplitude in the complex energy plane much easier compared to the exponential form, where simple singularities of the eikonal function would lead to the essential singularities in the amplitude. In potential scattering the eikonal (exponential) and U–matrix (rational) forms of unitarization correspond to two different approximations of the scattering wave function, which satisfy the Schrödinger equation

to the same order [6]. Rational form of unitarization corresponds to an approximate wave function which changes both the phase and amplitude of the wave. This form follows from dispersion theory. It can be rewritten in the exponential form but with completely different resultant phase function, and relation of the two phase functions is given in [6]. The rational form of unitarization in quantum field theory is based on the relativistic generalization [7] of the Heitler equation of radiation dumping [8]. In this approach an elastic scattering amplitude (we consider scattering of spinless particles for the moment) is a solution of the following equation in the c.m.s.

$$F(\mathbf{p},\mathbf{q}) = U(\mathbf{p},\mathbf{q}) + i\frac{\pi}{8}\rho(s)\int d\Omega_{\hat{\mathbf{k}}} U(\mathbf{p},\mathbf{k})F(\mathbf{k},\mathbf{q}), \qquad (1)$$

where $\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$ and $\mathbf{q} = \mathbf{q}_1 = -\mathbf{q}_2$ are momenta of the initial and final particles. The kinematical factor $\rho(s) \simeq 1$ at $s \gg 4m^2$ and will be neglected in the following. The equation (1) has simple solution in the impact parameter representation[1]:

$$(F,U)(s,t) = i\frac{s}{\pi^2}\int_0^\infty bdb(f,u)(s,b)J_0(b\sqrt{-t}),$$

i.e.

$$f(s,b) = \frac{u(s,b)}{1+u(s,b)}. \qquad (2)$$

Eq. (1) allows one to fulfill the unitarity provided the inequality

$$\mathrm{Re}\, u(s,b) \geq 0 \qquad (3)$$

is satisfied[2] . The inelastic overlap function,

$$\eta(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{inel}}{db^2},$$

i.e. the sum of all inelastic channel contributions into unitarity equation

$$\mathrm{Re}\, f(s,b) = |f(s,b)|^2 + \eta(s,b), \qquad (4)$$

[1]We factored out here an imaginary unity to provide a more compact form for the helicity amplitudes in what following.

[2]This is the only requirement needed to get an amplitude limited by unity $|f(s,b)| \leq 1$ (as unitarity requires), the function $u(s,b)$ itself should not obey such constraint.

has the following expression in terms of the function $u(s,b)$:

$$\eta(s,b)=\frac{\mathrm{Re}u(s,b)}{|1+u(s,b)|^2}. \tag{5}$$

The function $U(s,t)$ is the generalized reaction matrix, which is considered to be an input dynamical quantity similar to eikonal function. In potential scattering this function is related to the potential [6], i.e.

$$u(s,b)\sim\int_{-\infty}^{\infty}dzV(\sqrt{z^2+b^2}).$$

Construction of the particular models for the relativistic case in the framework of the U–matrix approach proceeds the common steps, i.e. the basic dynamics as well as the notions on hadron structure being used to obtain a particular form for the U–matrix. It is interesting to note that the form for the scattering amplitude analogous to Eq. (2) was obtained by Feynman in his parton model for diffractive scattering (which he has never published, cf. [9]).

In what follows we will not use a model features and detailed structure of $u(s,b)$, but consider reasonable arguments of a general nature, e.g. for the function $u(s,b)$ we can adopt a simple form

$$u(s,b)=gs^{\Delta}e^{-\mu b}, \tag{6}$$

where the parameter $\Delta>0$ guarantees the rise of the total cross-section. This is a rather general parameterization for $u(s,b)$ which provides correct analytical properties in the complex t–plane, i.e. it is consistent with the representation for the function $u(s,b)$:

$$u(s,b)=\frac{\pi^2}{is}\int_{t_0}^{\infty}\omega(s,t)K_0(b\sqrt{t})dt. \tag{7}$$

The Eq. (7) is a Fourier–Bessel transform of the spectral representation for the U–matrix[3]:

$$U(s,t)=\int_{t_0}^{\infty}\frac{\omega(s,t')}{t'-t}dt', \tag{8}$$

where the function $\omega(s,t)$ is the corresponding discontinuity of the function $U(s,t)$ [10].

[3]In fact, it is valid separately for its even and odd parts regarding cosine of the scattering angle.

Equation (1) for the helicity amplitudes of pp–scattering (i.e. for the two–fermion scattering) has the following form in the c.m.s. [11]:

$$\begin{aligned} F_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(\mathbf{p},\mathbf{q}) &= U_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(\mathbf{p},\mathbf{q}) + \\ & i\frac{\pi}{8}\sum_{\lambda',\lambda''}\int d\Omega_{\hat{\mathbf{k}}} U_{\lambda_3,\lambda_4,\lambda',\lambda''}(\mathbf{p},\mathbf{k}) F_{\lambda',\lambda'',\lambda_1,\lambda_2}(\mathbf{k},\mathbf{q}), \end{aligned} \tag{9}$$

where $\lambda' s$ are the initial and final proton's helicities. F_i are the helicity amplitudes in the standard notations, i.e.

$$F_1 \equiv F_{1/2,1/2,1/2,1/2},\; F_2 \equiv F_{-1/2,-1/2,1/2,1/2},\; F_3 \equiv F_{1/2,-1/2,1/2,-1/2}$$

and

$$F_4 \equiv F_{1/2,-1/2,-1/2,1/2},\; F_5 \equiv F_{1/2,1/2,1/2,-1/2}.$$

In the impact parameter representation for the helicity amplitudes F_i and the helicity functions U_i:

$$(F,U)_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(s,t) = i\frac{s}{\pi^2}(-1)^{N-\lambda}\int_0^\infty bdb (f,u)_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(s,b) J_{|\lambda_1-\lambda_2-\lambda_3+\lambda_4|}(b\sqrt{-t}),$$

where $N \equiv \min[(\lambda_1-\lambda_2),(\lambda_3-\lambda_4)]$, $\lambda \equiv \lambda_1 - \lambda_2$, we will have a system of the algebraic equations

$$f_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(s,b) = u_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(s,b) - \sum_{\lambda',\lambda''} u_{\lambda_3,\lambda_4,\lambda',\lambda''}(s,b) f_{\lambda',\lambda'',\lambda_1,\lambda_2}(s,b). \tag{10}$$

Explicit solution of Eqs. (10) then has the following form:

$$\begin{aligned} f_1 &= \frac{(u_1+u_1^2-u_2^2)(1+u_3+u_4)-2(1+2u_1-2u_2)u_5^2}{(1+u_1-u_2)[(1+u_1+u_2)(1+u_3+u_4)-4u_5^2]}, \\ f_2 &= \frac{u_2(1+u_3+u_4)-2u_5^2}{(1+u_1-u_2)[(1+u_1+u_2)(1+u_3+u_4)-4u_5^2]}, \\ f_3 &= \frac{(u_3+u_3^2-u_4^2)(1+u_1+u_2)-2(1+2u_3-2u_4)u_5^2}{(1+u_3-u_4)[(1+u_1+u_2)(1+u_3+u_4)-4u_5^2]}, \\ f_4 &= \frac{u_4(1+u_1+u_2)-2u_5^2}{(1+u_3-u_4)[(1+u_1+u_2)(1+u_3+u_4)-4u_5^2]}, \\ f_5 &= \frac{u_5}{(1+u_1+u_2)(1+u_3+u_4)-4u_5^2}, \end{aligned} \tag{11}$$

where for simplicity we omitted in the functions $f_i(s,b)$ and $u_i(s,b)$ their arguments. Unitarity requires that $\mathrm{Re}u_{1,3}(s,b)\geq 0$, but the absolute values of the functions $u_i(s,b)$ should not be limited by unity. For the functions $u_{2,4}(s,b)$ we adhere to a simple general form similar to the above Eq. (6) (using arguments based on the analytical properties in the complex t–plane):

$$u_2 \sim u_4 \sim s^{\Delta} e^{-\mu b}. \tag{12}$$

To get an upper bound for the amplitudes $F_{2,4}(s,t)$ we consider the case when $u_{2,4}(s,b)$ are dominating ones. Then we have for the amplitudes $F_{2,4}(s,t)$ the following representation

$$F_2(s,t) = \frac{is}{\pi^2}\int_0^\infty bdb \frac{u_2(s,b)}{1-u_2^2(s,b)} J_0(b\sqrt{-t}) \tag{13}$$

and

$$F_4(s,t) = \frac{is}{\pi^2}\int_0^\infty bdb \frac{u_4(s,b)}{1-u_4^2(s,b)} J_2(b\sqrt{-t}) \tag{14}$$

Using for $u_{2,4}(s,b)$ the functional dependence in the form of Eq. (6) it can be shown that the amplitude $F_2(s,t=0)$ cannot rise faster than $s\ln s$ at $s\to\infty$ and the function

$$\hat{F}_4(s,t=0) \equiv [\frac{m^2}{-t}F_4(s,t)]|_{t=0}$$

cannot rise faster than $s\ln^3 s$ at $s\to\infty$.

Thus, we can state that the explicit account of unitarity in the form of U - matrix approach leads to the following upper bound for the cross-section difference

$$\Delta\sigma_T \leq c\ln s,$$

where

$$\Delta\sigma_T \equiv \sigma_{tot}(\uparrow\downarrow) - \sigma_{tot}(\uparrow\uparrow) \sim -\frac{1}{s}\mathrm{Im}F_2(s,t=0).$$

It should be noted that the asymptotic behaviour of the amplitudes F_1 and F_3 are determined by the functions u_2 and u_4, respectively, in the situation when these functions dominate; the Froissart–Martin asymptotical bound for these amplitudes remains under these circumstances, i.e. they are limited by $cs\ln^2 s$ at $t=0$.

Another related important consequence is the conclusion on the possibility to neglect helicity-flip amplitudes F_2, F_4 and F_5 under calculations of differential cross-section

$$\frac{d\sigma}{dt} = \frac{2\pi^5}{s^2}(|F_1(s,t)|^2 + |F_2(s,t)|^2 + |F_3(s,t)|^2 + |F_4(s,t)|^2 + 4|F_5(s,t)|^2)$$

and double helicity-flip amplitudes F_2 and F_4 under calculation of analyzing power A_N

$$A_N(s,t)\frac{d\sigma}{dt} = \frac{2\pi^5}{s^2}\mathrm{Im}[(F_1(s,t) + F_2(s,t) + F_3(s,t) - F_4(s,t))^* F_5(s,t)]$$

in the region of small values of t in high energy limit. This conclusion is based on the above bounds for the helicity amplitudes and their small t dependence due to angular momentum conservation, i.e. at $-t \to 0$: $F_i \sim$ const, $(i = 1,2,3)$, $F_5 \sim \sqrt{-t}$ and $F_4 \sim -t$. However, the dominance of the helicity-non-flip amplitudes ceases to be valid at fixed values of momentum transfers, where , e.g. amplitude F_4 can become a dominant one, since its energy growth is limited by the function $s\ln^3 s$, while other helicity amplitudes cannot increase faster than $s\ln^2 s$.

One should recall that unitarity for the helicity amplitudes leads to a peripheral dependence of the amplitudes $f_i(s,b)$ $(i = 2,4,5)$ on the impact parameter b at high energy, i.e.

$$|f_i(s,b=0)| \to 0$$

at $s \to \infty$. This is a consequence of the explicit unitarity representation for the helicity amplitudes through the U-matrix and it is this fact allows one to get better bounds for the helicity-flip amplitudes.

Thus, as it was shown in this note and in [4], we have the following asymptotic results:

- the ratio $r_5(s,0) \equiv 2\hat{F}_5(s,0)/[F_1(s,0) + F_3(s,0)]$ cannot increase with energy,
- the amplitude $F_2(s,t=0)$ cannot increase faster than $s\ln s$,
- the function $\hat{F}_4(s,t=0)$ should not rise faster than $s\ln^3 s$ at high energies.

Nowadays RHIC spin program includes experiments with two polarized proton beams at the highest available energies and the above bounds could be useful and provide grounds for the estimations of the spin observables in the forward region in these experiments. The above bounds provide justification of the smallness of the double helicity-flip amplitudes in the low-t region, but simultaneously they imply an importance of the double helicity-flip amplitudes at the moderate values of momentum transfers. This result is in accordance with early analysis of experimental data performed in [5]. Magnitude of the helicity amplitude F_2 at $t = 0$ can be measured directly at RHIC through the measurements of $\Delta\sigma_T$ [12] and it

is definitely an important study of the spin properties of diffraction. The experimental data for $\Delta\sigma_T(s)$ could also be a useful source of information on the low-x behaviour of the spin structure function $h_1(x)$.

References

[1] A. T. Bates and N. H. Buttimore, Phys. Rev. D 65, 014015 (2001) and references therein;

[2] B. Z. Kopeliovich, Plenary talk at 15th International Spin Physics Symposium (SPIN 2002), Long Island, New York, 9-14 Sep 2002, hep-ph/0211061.

[3] N. H. Buttimore, B. Z. Kopeliovich, E. Leader, J. Soffer and T. L. Trueman, Phys. Rev. D 59, 114010 (1999).

[4] S. M. Troshin and N.E. Tyurin, Phys. Lett. B 459, 641 (1999);

[5] V. F. Edneral, S. M. Troshin and N. E. Tyurin, JETP Lett. 30, 330 (1979);

[6] R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).

[7] A. A. Logunov, V. I. Savrin, N. E. Tyurin and O. A. Khrustalev, Teor. Math. Phys. 6, 157, (1971).

[8] W. Heitler, Proc. Cambr. Phil. Soc., 37, 291 (1941).

[9] F. Ravndal, "Feynman's (secret?) parton model for diffraction scattering", contributed paper presented at the Conference QCD — 20 Years Later, Aachen, June 9-13, 1992; further discussions of this paper and model can be found in S. Barshay, P.Heiliger and D. Rein, Mod. Phys. Lett. A 7, 2559 (1992).

[10] V.I. Savrin and N.E. Tyurin, Theor. Math. Phys. 23, 561 (1976).

[11] S.M. Troshin and N.E. Tyurin, Fiz. Elem. Chast. Atom. Yadra 19, 997 (1988); Phys. Usp. 37, 991 (1994).

[12] W. Guryn *et al.*, RHIC Proposal R7 (1994) (unpublished); V. Kanavets, Czech. J. Phys., Suppl. A, **53**, A21 (2003).